Confidence Interval for a Proportion

We want to estimate a population proportion p (binomial probability of success).

Point Estimate $\pm$ Margin of Error   (Margin of Error is also called Error Bound)

Point Estimate: Sample Proportion: $p' = \frac{x}{n} = \frac{number\ of\ successes\ in\ sample}{total\ number\ in\ sample}$

Error Bound: EBP = $Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$

$Z_{\alpha/2}$ is the Z value that will put to an area equal to the confidence level (CL) in the middle between $\pm Z_{\alpha/2}$, using the standard normal distribution N(0,1)

$Z_{\alpha/2}$ tells us how many appropriate standard deviations to enclose about the point estimate

$\sqrt{\frac{p'q'}{n}}$ is the appropriate standard deviation for proportions

Confidence Interval: $p' \pm EBP$ which is $p' \pm Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$

Confidence Interval for a Mean μ when σ is known

We want to estimate the population average μ and we already know the population standard deviation σ.

Point Estimate $\pm$ Margin of Error   (Margin of Error is also called Error Bound)

Point Estimate: Sample Average (Sample Mean) $\bar{x}$

Error Bound: EBM = $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$Z_{\alpha/2}$ is the Z value that will put to an area equal to the confidence level (CL) in the middle between $\pm Z_{\alpha/2}$, using the standard normal distribution N(0,1)

$Z_{\alpha/2}$ tells us how many appropriate standard deviations to enclose about the point estimate

$\frac{\sigma}{\sqrt{n}}$ is the appropriate standard deviation for the sample average (sample mean)

Confidence Interval: $\bar{x} \pm EBM$ which is $\bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence Interval for a Mean μ when σ is NOT known

We want to estimate the population average μ and we do not know the population standard deviation σ.
We use the sample standard deviation s to estimate the population standard deviation σ

Point Estimate $\pm$ Margin of Error   (Margin of Error is also called Error Bound)

Point Estimate: Sample Average (Sample Mean) $\bar{x}$

Error Bound: EBM = $t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$t_{\alpha/2}$ is the t value that will put an area equal to the confidence level (CL) in the middle between $\pm t_{\alpha/2}$ using the student t-distribution with df = n−1

$t_{\alpha/2}$ tells us how many standard deviations we need to move away from the point estimate

$\frac{s}{\sqrt{n}}$ is an estimate of the appropriate standard deviation for the sample average (sample mean)

Confidence Interval: $\bar{x} \pm EBM$ which is $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

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Interpreting the Confidence Interval:

For a proportion: ***2 ways to word it***

We are _____% confident that the true proportion of the population that
describe the random variable in the situation of this problem is between ______ and ________

We are _____% confident that between _______% and _______% of the population
describe the random variable in the situation of this problem

For a mean (average)

We are _____% confident that the true population average (or mean) *describe the random variable in the situation of this problem* is between _______ and _________

What does it mean to be CL% confident:

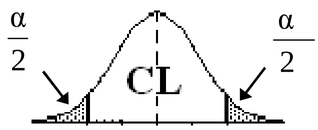
If we were to take repeated samples and calculate many confidence interval estimates based on those samples, then we would expect that CL% of the confidence interval estimates would be "good estimates" that would enclose (capture) the true value of the population parameter we are trying to estimate.

If we were to take repeated samples and calculate many confidence interval estimates based on those samples, then we would expect that 100% − CL% of the confidence interval estimates would be "bad estimates" that would NOT enclose (capture) the true value of the population parameter we are trying to estimate.

Note that the confidence interval is about proportions or averages. It is not about individual data values. It does not mean that CL% of the data lies within the confidence interval.

To find Z that puts the area equal to the confidence level "in the middle"

CL is in the middle
$\alpha = 1 - CL$ is "outside", split between both tails
$\frac{\alpha}{2}$ is in one tail.

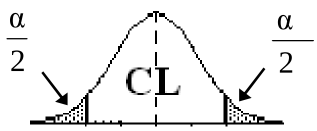


To find $Z_{\alpha/2}$: invnorm($1-\frac{\alpha}{2}$, 0, 1) ; *OR* invnorm($\frac{\alpha}{2}$, 0, 1) use absolute value (drop the "−" sign)

Without calculator: Use a standard normal probability table to find Z.

To find t that puts the area equal to the confidence level "in the middle"

CL is in the middle
$\alpha = 1 - CL$ is "outside", split between both tails
$\frac{\alpha}{2}$ is in one tail, df = degrees of freedom = n−1



To find $t_{\alpha/2}$: TI-84+: invT($1-\frac{\alpha}{2}$, df) ; *OR* invT ($\frac{\alpha}{2}$, df) use absolute value (drop the "−" sign)

TI-83,83+: Use INVT program; get it downloaded to your calculator from instructor: PRGM INVT

put in area to the left and df after the prompts: area to left is $1-\frac{\alpha}{2}$; (if using $\frac{\alpha}{2}$, the "−" sign)

TI-86 or no calculator: Use a student's-t distribution probability table. The t value is found at the intersection of the column for the appropriate confidence level and the row for degrees of freedom

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EXAMPLE 1: **CONFIDENCE INTERVAL ESTIMATE for an unknown POPULATION PROPORTION p**

a. A city government needs to determine the percent of its residents that do not have health insurance coverage. The city health department conducts a survey of 1600 city residents and finds that 244 of the 1600 residents included in the survey do not have health insurance. Construct and interpret a 95% interval for the true population proportion of all city residents who do not have health insurance. Use a 95% confidence level.

p = ____

p′ = ____

We are using sample data to estimate an unknown proportion for the whole population

Point Estimate = p′	EBP = $Z_{\alpha/2}\sqrt{\frac{p'q'}{n}}$	Confidence Interval p′ ± EBP	$Z_{\alpha/2}$ = invnorm(1−$\frac{\alpha}{2}$, 0, 1) *α = 1 – Confidence Level*

a. Calculations and interpretation in context of the problem

b. It has been estimated that for the state in which that city is located, approximately 20% of residents do not have health insurance. Based on this confidence interval, can we conclude that the proportion of city residents who lack health insurance is lower than the proportion of state residents? Explain.

c. It has been estimated that nationally, approximately 16% of residents do not have health insurance. Based on this confidence interval, can we conclude that the proportion of city residents who lack health insurance is lower than the proportion of U.S. residents? Explain.

d. What does it mean when we say that the confidence level is 95% or that we are "95% confident"?

EXAMPLE 2: CONFIDENCE INTERVAL ESTIMATE for unknown POPULATION MEAN μ when the POPULATION STANDARD DEVIATION σ is KNOWN

Suppose that a soda bottling plant fills 12 ounce cans with soda. However the filling machine varies and does not fill each can with exactly 12 ounces. To determine if the filling machine needs adjustment, each day the quality control manager measures the amount of soda per can for a random sample of fifty cans. Experience shows that its filling machines have a known standard deviation of 0.35 ounces. In today's sample of 50 cans of soda, the average amount of soda per can is 12.1 ounces. Construct and interpret a 95% confidence interval estimate for the true population average amount of soda contained in all cans filled today at this bottling plant.

μ = ____

$\bar{x}$ = ____

We are using sample data to estimate an unknown mean (average) for the whole population

Point Estimate = $\bar{x}$	EBM = $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	Confidence Interval $\bar{x} \pm$ EBM	$Z_{\alpha/2}$ = invnorm(1−$\frac{\alpha}{2}$, 0, 1) *α = 1 – Confidence Level*

EXAMPLE 3: CONFIDENCE INTERVAL ESTIMATE for unknown POPULATION MEAN μ when the POPULATION STANDARD DEVIATION σ is <u>NOT KNOWN</u>

a. The speeds of 20 vehicles are observed by radar on a particular road.
For the vehicles in the sample, the average speed is 31.3 miles per hour with standard deviation 7.0 mph.
Construct and interpret confidence interval estimate of the true population average speed of all vehicles traveling on this road. Use a 90% confidence level.

μ = ____

$\bar{x}$ = ____

We are using sample data to estimate an unknown mean (average) for the whole population

Point Estimate = $\bar{x}$	EBM = $t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	Confidence Interval $\bar{x} \pm$ EBM	TI-84: $t_{\alpha/2}$ = invT(1−$\frac{\alpha}{2}$, df) TI-83: use InvT program *α = 1 – Confidence Level df = n-1*

b. In Example 2, suppose that instead of you were not given the sample mean and sample standard deviation and instead you were given a list of data for the speeds (in miles per hour) of the 20 vehicles:

19	19	22	24	25	27	28	30	31	33
35	37	30	37	36	39	40	43	36	35

How would you use the data to do this problem?