

CONFIDENCE INTERVAL ESTIMATE FOR AN UNKNOWN POPULATION PROPORTION

This example uses fictional data – I did not really do this, but it would be interesting.

On the Friday before a long weekend, the Dean wanted to know what proportion of all students with classes meeting on a Friday at a community college actually attended class. A statistics instructor randomly selected a sample of 250 students who were enrolled in classes that meet on Fridays and by checking attendance with instructors was able to determine that 163 of the 250 students attended class.

Construct and interpret a 90% confidence interval estimate of the true proportion of all students at the college who attended class on the Friday before a long weekend.

p = true population proportion of all students who attended class (UNKNOWN)

p' = sample proportion of all students who attended class (KNOWN from sample data)

We are using sample data to estimate an unknown proportion for the whole population

p' is Point Estimate of p :
$$p' = \frac{x}{n} = \frac{163}{250} = 0.652$$

Z Score

Find Z that puts an area of 0.90 (this is the confidence level) in the middle of $Z \sim N(0,1)$

$\text{invnorm}(0.05, 0, 1) = -1.645$ (Accompanying graph was drawn in class; graphic not available)

Use $Z = 1.645$

Error Bound
$$EBP = Z \sqrt{\frac{p'q'}{n}} = 1.645 \sqrt{\frac{0.652 * 0.348}{250}} = 1.645(0.0301) = 0.0496$$

Note 1: The “error bound” is the “margin of error”

Note 2:
$$\sqrt{\frac{p'q'}{n}} = \sqrt{\frac{0.652 * 0.348}{250}} = 0.0301$$
 is called the “standard error” of the proportion

It is the **appropriate standard deviation** to use **for proportions**

Confidence Interval: $p' \pm EBP$
$$p' \pm Z \sqrt{\frac{p'q'}{n}}$$

$$p' - EBP = 0.652 - 0.0496 = 0.6024$$

$$p' + EBP = 0.652 + 0.0496 = 0.7016$$

$$(0.6024, 0.7016)$$

Interpretation

We are 90% confident that between 60.24% and 70.16% of all students at this college, who have classes that meet on Friday, actually attended class on the Friday before a long weekend.

Other Possible Wordings of the Interpretation

We are 90% confident that the true population proportion of all students at this college who have classes that meet on Friday that actually attended class on the Friday before a long weekend is between 60.24% and 70.16%

For all students at this college who are enrolled in classes that meet on Friday, We are 90% confident that between 60.24% and 70.16% of all such students actually attended classes on the Friday before a long weekend.

Note 3: For the SAMPLE, the proportion who attended is exactly 65.2% - we know the data.

For the POPULATION of all students we do not know the true attendance data.

We used the sample data to estimate that the population proportion is between 60.24% and 70.16%

We used an interval estimate because we don't really believe that the true population proportion is exactly the same that as the sample data, but we believe it is close to the sample data.